Journal of Pure and Applied Algebra 36 (1985) 217
North-Holland

NOTE

A NOTE ON ULTRAPRODUCTS OF COMPLETE BOOLEAN ALGEBRAS

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Communicated by H. Bass
Received November 1984

In this Note we provide an example that corrects the statement made in the last sentence of Remark 1.3 of [1].

Example. With all notation as in [1], let K denote the first measurable cardinal. For all $\alpha < K$, let us take the boolean algebra B_α to be $\mathscr{P}(\alpha)$. In this set-up, there exists a K-complete non-principal ultrafilter $\mathscr{U}$ over K such that the ultraproduct $\mathscr{B} = \prod_{\alpha<K} B_\alpha/\mathscr{U}$ is $\mathscr{P}(K)$, the power set of K, and hence complete.

Such an ultrafilter is called *normal.*

Let us observe that the degree of completeness of $\mathscr{U}$ is K, that the cardinality of B_α approaches K since K as a measurable cardinal is not accessible and therefore is the limit of the α's, $\alpha<K$, and yet the ultraproduct is complete.

The key result needed to verify the completeness of $\mathscr{B}$ is as follows: If one has a countably complete ultrafilter of degree of completeness K, then every subset of the ultraproduct of cardinality at most K is the universe of some ultraproduct of sets.

The normality of $\mathscr{U}$ gives the fact that $\mathscr{B}$ has exactly K atoms.

The reader can consult [2] for the above mentioned results about measurable cardinals.

References

[1] M. Contessa, Ultraproducts of pm-Rings and mp-Rings, J. Pure Applied Algebra 32 (1984) 11-20.
[2] T. Jech and K. Hrbacek, Introduction to Set Theory (Marcel Dekker, New York, 1978).